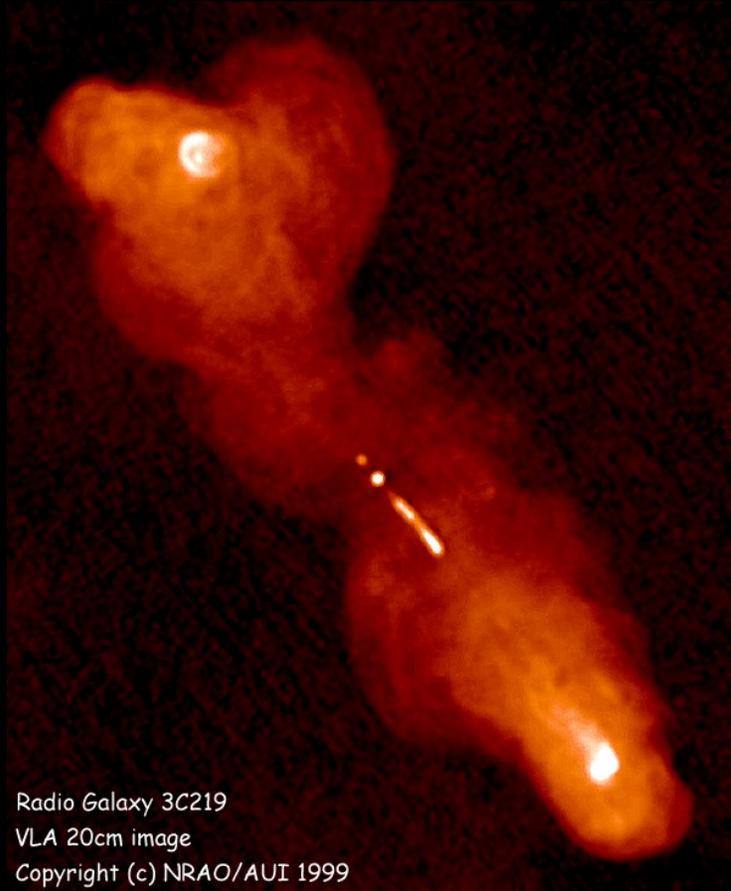


**The Magneto-Rotational Instability
and turbulent
angular momentum transport**

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Paul Fischer
Aleksandr Obabko**



Radio Galaxy 3C219
VLA 20cm image
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300 Kp

Accretion onto a central compact object is believed to power some of the most energetic phenomena in the universe

Black hole accretion (Lynden-Bell 1969)

- Central mass $\sim 10^8 - 10^{10} M_{\odot}$
- Accretion rate $\sim 1 M_{\odot}/\text{yr}$
- Total luminosity $\sim 10^{47} L_{\odot}$

Angular momentum transport

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- If angular momentum is conserved matter just orbits the central object
- Accretion rate is determined by the outward transport of angular momentum



- Frictional or viscous transport too inefficient to explain observed luminosities
- Something many orders of magnitude more efficient is needed

Turbulent transport

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Shakura & Sunyaev (1973) assumed transport was due to turbulence in the disc. For reasonable transport rates assumption gave

Reasonable disc structures

Reasonable accretion rates

In a turbulent flow effective diffusivity

$$D \approx U\ell = \nu \left(\frac{U\ell}{\nu} \right) = \nu \text{Re}$$



characteristic length scale

characteristic velocity

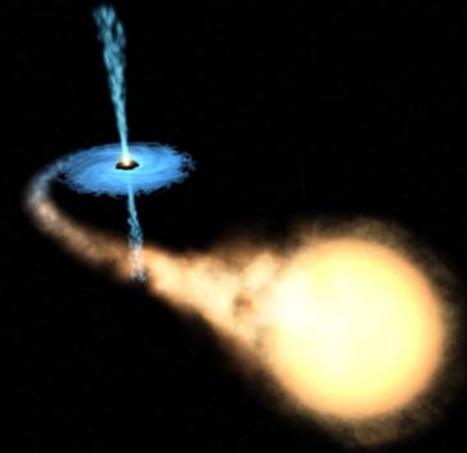
What is the physical origin of the turbulence?

Keplerian discs

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Most astrophysical discs are close to Keplerian

- nearly circular
- angular velocity profile $\Omega \propto r^{-3/2}$
 - angular velocity increases inwards
 - angular momentum ($r^2\Omega$) increases outwards



Stable to axi-symmetric
disturbances

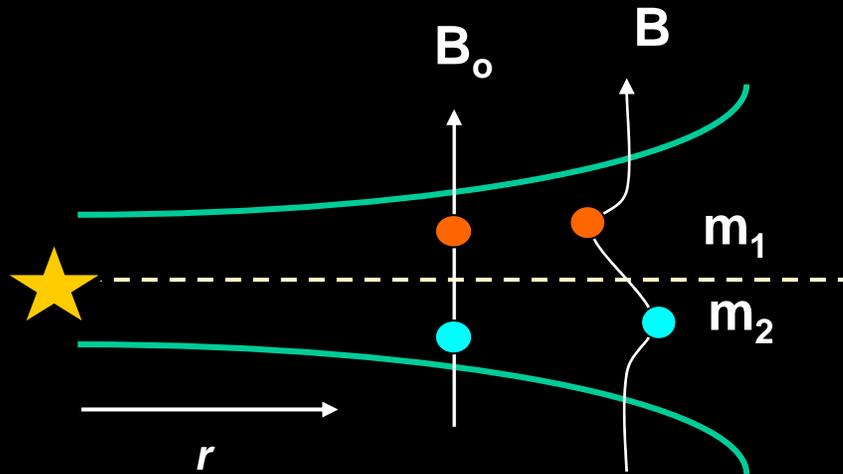
(Rayleigh criterion)



The Magneto-Rotational Instability

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Stability changes dramatically if disc is even weakly magnetized
(Velikov 1959; Balbus & Hawley 1991)



- New instability criterion is that angular velocity increases inwards
- Effect of instability is to transport angular momentum outwards

Laboratory experiments

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- **Instability depends on velocity profile, not gravitational force**
- **Can be studied in Couette flow between concentric cylinders using liquid metals (Na, Ga)**
- **Spin cylinders so that basic state has circular streamlines and**

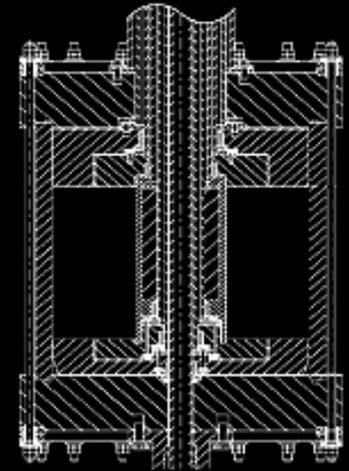
$$\frac{d\Omega}{dr} \leq 0 \quad \text{and} \quad \frac{d(r^2\Omega)}{dr} > 0$$



New Mexico exp: S. Colgate



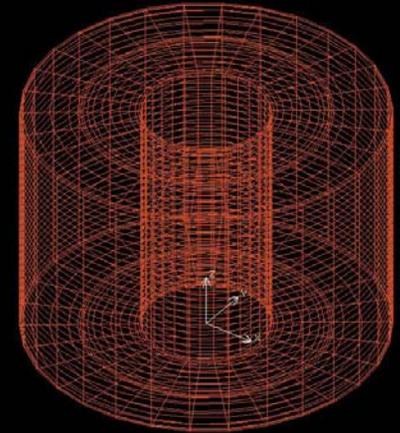
Princeton exp: J. Goodman & H. Ji



Three basic questions

- **What is the effect of the endplates?**
 - secondary Ekman circulation
 - Stewartson layers
- **Can the MRI be demonstrated?**
 - basic state not purely Couette flow
 - enhanced angular momentum transport due to Ekman turbulence
- **What does the nonlinear state look like?**
 - flow visualization in liquid metals difficult
 - what contributes to the angular momentum flux?

- **Solve incompressible MHD equations for a viscous, electrically conducting fluid**
- **Cylindrical geometry with different endplates**
 - periodic
 - lids (same Ω as outer cylinder)
 - rings (two: rotating at intermediate Ω 's)
- **Use spectral-elements method optimized for highly parallel machines (based on Nekton 5000)**
- **Vary magnetic field strength**

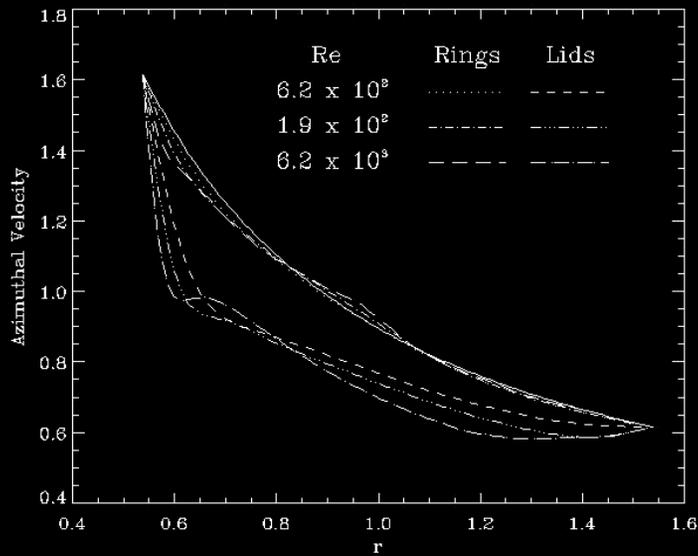


End effects: simulations

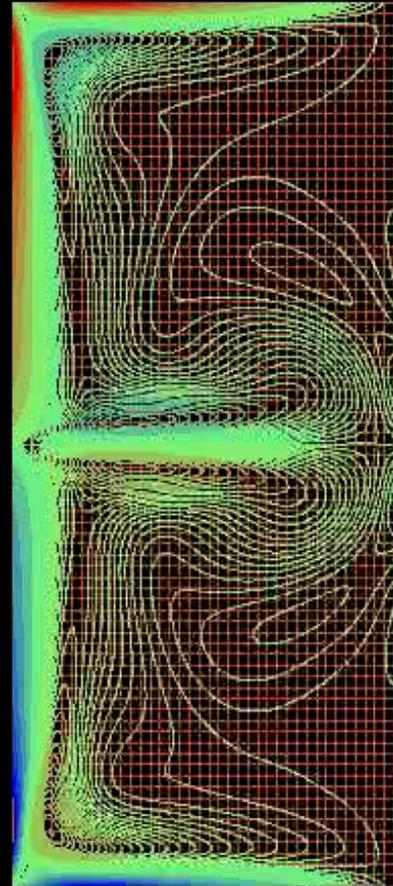
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Configurations:

- Periodic
- Lids
- Rings
-



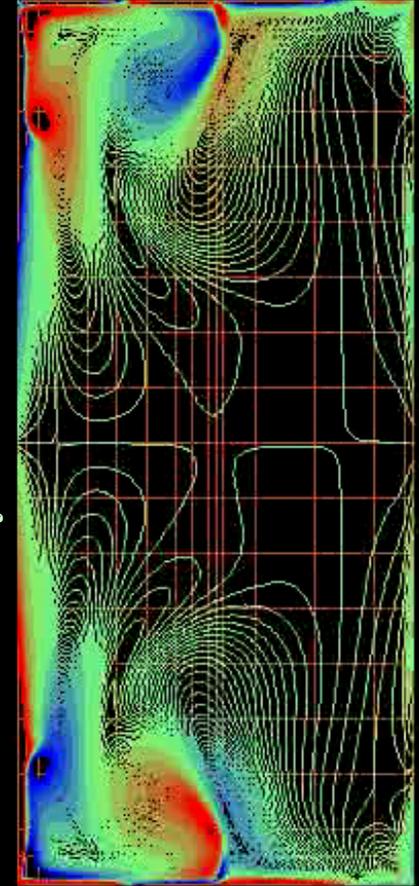
inner cylinder



lids

vorticity

inner cylinder

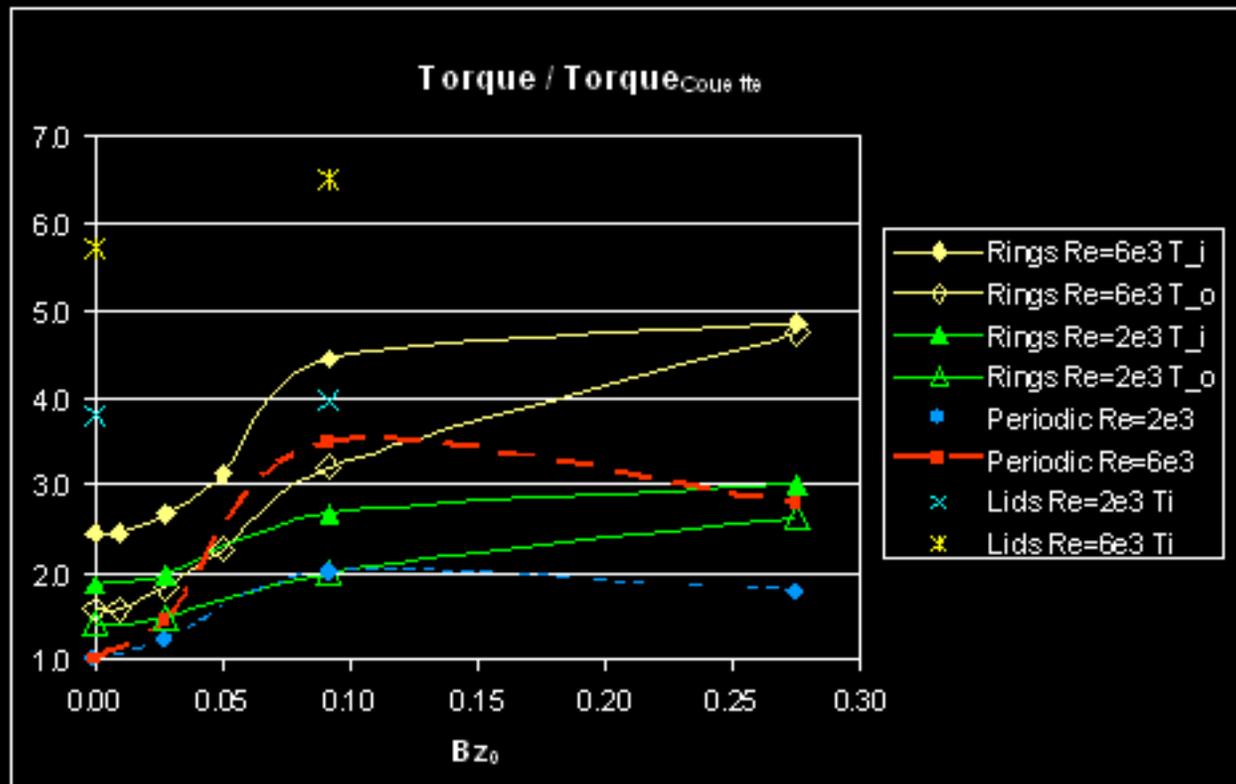


rings

Torque measurements: simulations

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- **Instability enhances angular momentum transport**
- **To keep same rotation rate torque on cylinders must be increased**

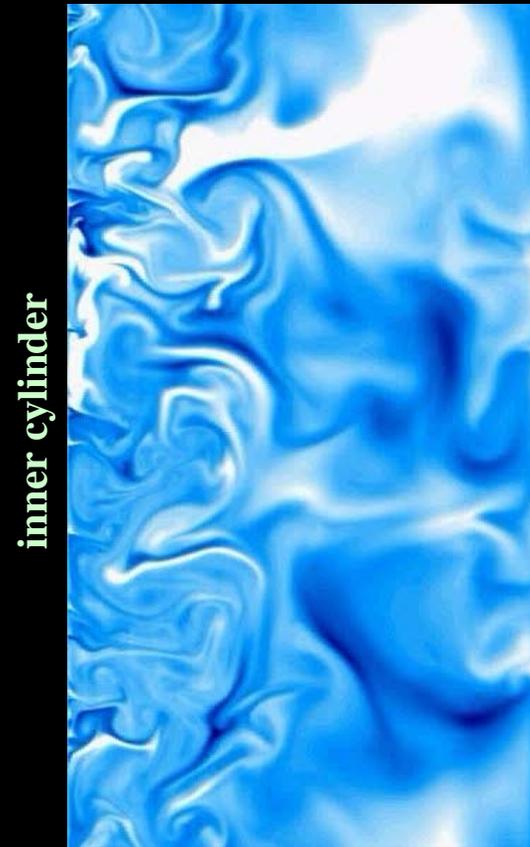
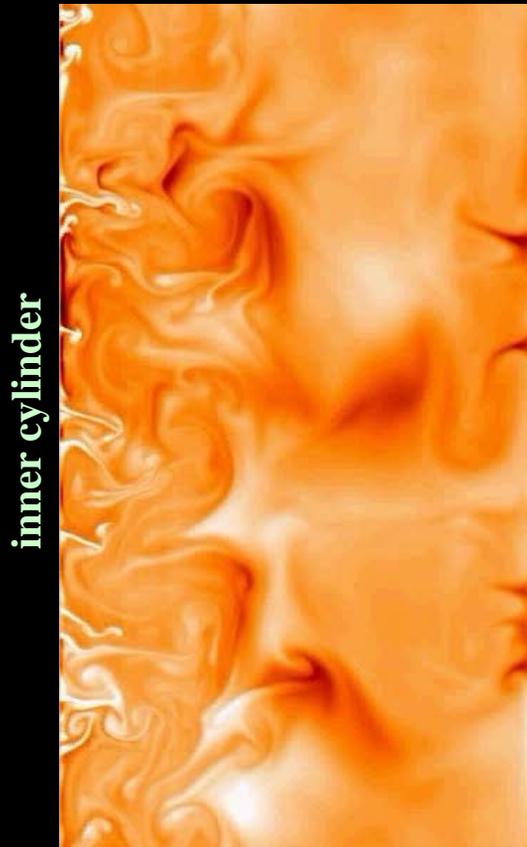


↑ applied magnetic field strength

Flow structure: simulations

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azimuthal fluctuations



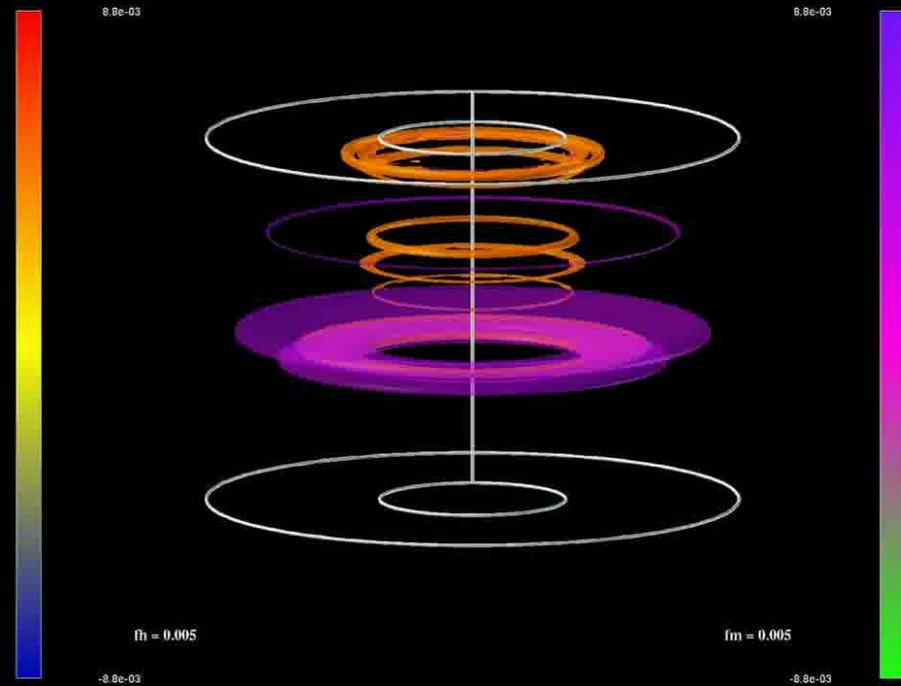
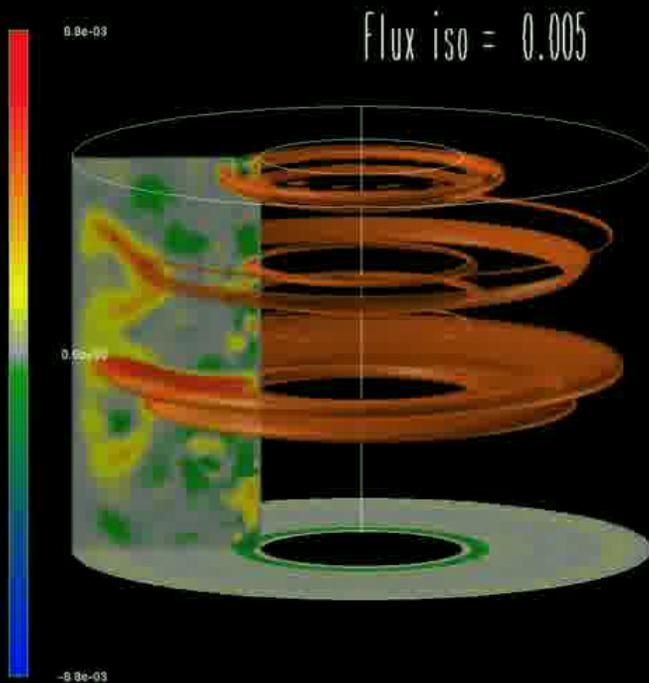
- Magnetic field expelled to outer regions
- Inner part dominated by eddy motions – plumes
- Outer part dominated by waves (magneto-inertial)

Angular momentum flux: simulations

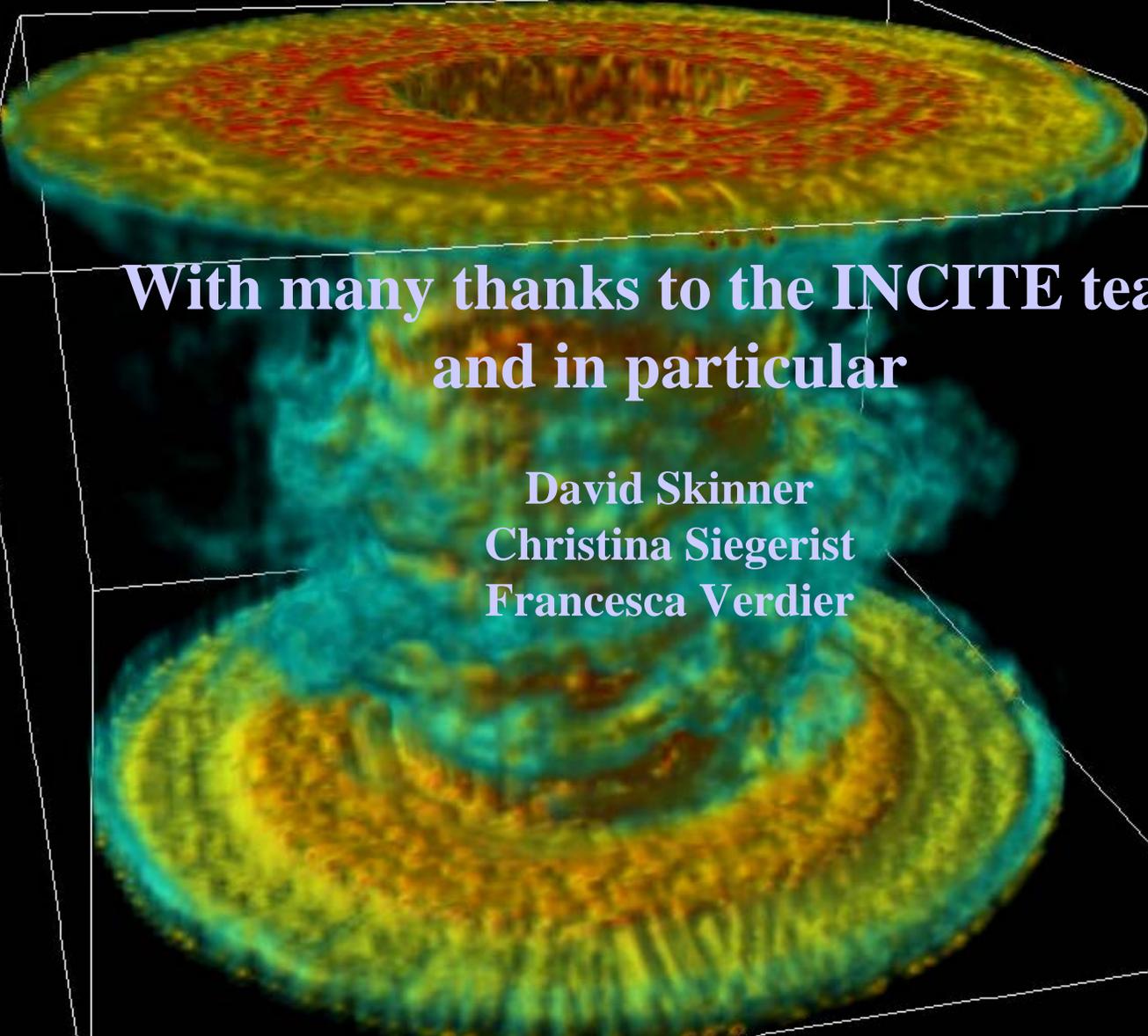
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$$F_{rz} = r \left[(u_\theta - \bar{u}_\theta) u_r - B_\theta B_r \right]$$

- **Transport by coherent structures (cf. convection)**
- **Reynolds stresses dominant in the inner part**
- **Maxwell stresses dominant in the outer part**



- **Using INCITE resources it is possible to demonstrate numerically the existence of the MRI in realistic (laboratory) geometry**
- **Numerical simulations complement experiment**
 - allow more flexible boundary conditions
 - can explore different parameter regimes (eg. high Rm limit)
 - afford superior flow visualization
- **Numerical simulations can guide design of future laboratory experiments**
- **Results of numerical simulation provide basis for formulation of phenomenological models of MRI turbulence and enhanced angular momentum transport**

A 3D visualization of a magnetic field structure, likely a tokamak configuration. It features two toroidal components, one at the top and one at the bottom, with a central column of magnetic field lines connecting them. The field lines are color-coded, with red and yellow indicating higher field strength and blue indicating lower field strength. The entire structure is enclosed in a white wireframe box.

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